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**Mathematics XXI Century & Natural Science**

**BOOK OF ABSTRACTS**

**IV International Seminar**

**Nonlinear Phenomenology Advances  
(October 3 – 6, 2017)**

**III International Workshop**

**”Scientific & Educational Problems of Belarusian  
Nuclear Power Plant Project”  
(October 4 – 6, 2017)**

**Edited by V. I. Antonov & D. W. Serow**

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The edition is intended for students, graduate students and researchers working in the field of application of nonlinear mathematic models and methods in various areas of natural science and technology.

Bibliography: 18 titles.

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## *Welcome to Nonlinear Phenomenology Advances: Mathematics XXI Century & Natural Science, 2017!*

*Dear Colleagues and Friends,*

*We are happy to welcome you in Saint Petersburg Hero City Leningrad at the opening ceremony of IV International Seminar "Nonlinear Phenomenology Advances: Mathematics XXI Century & Natural Science" including III International Seminar "Systemic-Operational Modifications of Mathematical Theories" and III International Workshop "Scientific & Educational Problems of Belarusian Nuclear Power Plant Project" organized by the Peter the Great St. Petersburg Polytechnic University.*

*Under the leadership of prominent scientists, during the period of 80 years the scientific schools in the field of natural sciences have been created in USSR to become the national property of our country. Of world renown are the studies of the Russian and Belarusian researchers in Nonlinear Science including such leads of research as nonlinear dynamics, wavelet analysis, number theory and cryptography, neural network modeling, soils nonlinear dynamics and evolution, self-organization and cooperative phenomena, spatiotemporal structures, turbulence, patterns in complex media, high energy physics et all. Ensuring continuity in science and advances of scientific commonwealth of Russian Academy of Sciences, National Academy of Sciences of Belarus, Belarusian State University, University of Beograd and Peter the Great St. Petersburg Polytechnic University are the recognized leaders in the field of Nonlinear Science. The fact that important international conferences attracting the scientists from numerous countries have been organized by our University is indicative of its generally recognized contribution to science and education. Presently, the Peter the Great St. Petersburg Polytechnic University is a large educational, research, innovative, production, and cultural center capable to influence the future of our country in many respects. Geography of the contacts maintained by the University involves all the continents, the majority of the European and numerous other countries. We welcome new contacts to solve a wide range of scientific problems in the field of Nonlinear Science, where the Russian researchers have a considerable forerun.*

*Dear participants of IV International Seminar "Nonlinear Phenomenology Advances: Mathematics XXI Century & Natural Science" we wish you new valuable ideas, fruitful research activities, strengthening of scientific contacts, and a pleasant stay in Saint Petersburg Hero City Leningrad.*

*Vice-Rector SPbPU  
Vitaly Sergeev*

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Abstracts

# Relation between Centrosome Excitation and Oscillatory Energy of Mitotic Spindle in Metaphase trough Biomechanical Oscillatory Model of Mitotic Spindle

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**Keywords:** chromosomes, centrosome, mitotic spindle, oscillations, energy, biomechanical model

Specific organization of metaphase chromosomes is due to complex molecular system of mitotic spindle. Chromosomes oscillate to align precisely during metaphase [1]. Positioning of centrosome, an organizer of microtubules in mitotic spindle, influence a directionality of cell division [2]. There are some opinions that functional genomic architecture is not only present in interphase but also in metaphase stage of cell division cycle [3]. The aim of this work was to study how different oscillatory behavior of centrosomes affects the energy of pairs of homologue chromosomes in the system of mitotic spindle. The analyses were done through mechanical oscillatory model of mitotic spindle [4]. In this model, mitotic spindle is presented as a system of coupled oscillators where one oscillatory pair consists of a centrosome, a microtubule and a related chromosome and these are interconnected with their homologous pairs. Centrosomes are presented as mass particles that represent two rheonomic centres of oscillations. Each element in the model has its mechanical counterpart: microtubules are standard light visco-elastic elements, homologue chromosomes are mass particles that are interconnected with standard light massless elastic spring.

Analytical expressions for potential and kinetic energy as well as for total mechanical energy of oscillating pair of homologues chromosomes are given. Numerical analysis with some approximation for human cell that carries X

chromosome is done. Influence of centrosomes' frequency on oscillatory energy of mitotic spindle is discussed. Total mechanical energy of oscillating pair of homologue chromosomes has oscillatory character. If rheonomic centres oscillate with different frequencies, energy of parts of mitotic spindle that belongs to two sister cells would differ. We suppose that this difference could be additional level of coding information that is transferred into the next cell generation and could be of interests in the process of cell differentiation.

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# Wada Basins and Finite von Karman Vortex Streets

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**Keywords:** dissipative dynamic system, multi-separatrices fixed point, Wada basins, von Karman vortex street, Birkhoff's curve, indecomposable continuum

Recently authors have constructed homogeneous dissipative dynamic systems acting on the plane with only fixed saddle and inverse saddle possessing *Wada property* [1, 2], i. e. there exist three regions being invariant with respect to dynamic system action (or double action in case of the inverse saddle) with common boundary (being Birkhoff's curve). The dynamic system action is defined to be formula

$$T \left( \frac{R\tilde{x} + \tilde{y}}{p} + iq\tilde{y} \right) e^{iR\varphi(\tilde{x}, \tilde{y})} \mapsto x + iy, \quad R, T \stackrel{def}{=} \pm 1,$$

where  $\varphi \stackrel{def}{=} \alpha \tanh^2 x + \beta \tanh^2 y$ ,  $p, q > 1$ ,  $\alpha, \beta > 0$  and  $r$  is equal -1 or 1. One is the model for Poincaré map for periodic second order differential equation

$$\ddot{\varphi} - (a - 3a\varphi^2)\dot{\varphi} + \varphi + 2\varphi^2 b \cos \omega t - \varphi^3 = 0.$$

*Wada basins* are called to be the a simply connected regions with compact closure having common boundary. It is clear that can exist at least two Wada basins.

The dynamic system action  $\psi_k \in \text{Diff}(\mathbb{E}^2)$  is defined to be formula

$$\alpha(\Re w(|\tilde{z}|, \arg \tilde{z}) + i\Im w(|\tilde{z}|, \arg \tilde{z}))e^{i\gamma|\tilde{z}|} \mapsto z, \quad z \stackrel{def}{=} x + iy, \quad \tilde{z} \stackrel{def}{=} \tanh x + i \tanh y$$

at iteration, where  $w(|\tilde{z}|, \arg \tilde{z}) \stackrel{def}{=} |\tilde{z}| (e^{i \arg \tilde{z}} + \lambda e^{-ni \arg \tilde{z}})$ ,  $n \in \mathbb{N}$ . The system is homogeneous in the neighborhood of the fix point  $(0, 0)$  being *multi-separatrices fixed point*. Thereby one is characterized stable manifold being

$n + 1$  stable separatrices union and unstable manifold being  $n + 1$  unstable separatrices union. Therefore Euler characteristics  $\chi(0, 0) = -n$  and there exist  $n + 1$  antisaddle fixed points.

**Theorem 1.** *There exists Birkhoff's curve  $\Upsilon_\psi$  being common boundary more then three regions, such that one contains multi-separatrices fixed point.*

The following dynamic system action  $\phi_k \in \text{Diff}(\mathbb{E}^2)$  is defined to be formulae

$$\begin{aligned} \text{Arg}((\Re w(|\tilde{z}|, \arg \tilde{z}) + i\Im w(|\tilde{z}|, \arg \tilde{z}))e^{i\gamma|\tilde{z}|}) &\mapsto x, \\ \alpha \cdot \sqrt{\Re^2 w(|\tilde{z}|, \arg \tilde{z}) + \Im^2 w(|\tilde{z}|, \arg \tilde{z})} &\mapsto y \end{aligned}$$

at iterations. It is clear that the system has  $n$  fixed saddles  $(2\pi l/n, 0)$ ,  $l = \overline{1, n}$  and  $n + 1$  fixed antisaddles.

**Theorem 2.** *There exists Birkhoff's curve  $\Upsilon_\phi$  being common boundary more then three regions, such that one contains more than one fixed anti-saddle.*

Thereby  $n$  Wada basins and ocean (regions with common boundary) form the *finite von Karman vortex street*.

**Theorem 3.** *Statements of theorems 1. and 2. are dual. Therefore  $\Upsilon_\psi$  and  $\Upsilon_\phi$  are dual.*

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# Invariant Measures of Dynamical Polysystems and Estimates of Rate of Convergence of Control Times in Discrete Time

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**Keywords:** dynamical polysystem, invariant measure,

Control systems with a finitely many of control settings, i. e. dynamical polysystems, are considered. Assuming that a polysystem is controllable in continuous time, we investigate its controllability in discrete time. Under the rank condition, the switching controls always exist. In this case, if control error is provided arbitrarily small then usually a control time is becoming arbitrarily large. Thus, there is a problem to study dependence of control time on target conditions and on control errors. It is shown that this dependence can be described by metric characteristics as follows. For an original polysystem, it is introduced an auxiliary polysystem which is called an associated polysystem. For this associated polysystem, it is established existence of a set of ergodic invariant measures. Using ergodic invariant measures, we define distribution functions of estimates of switching instants. In these terms, the rate of convergence of control times in discrete time is given. The estimates of control times can be find by numerical methods. Each distribution function describes a certain type of sequence of switching instants depending on target points. In addition, it is specified classes of systems with a typical sequences of switching instants. Typicalness of sequences means that properties of these sequences are the same for almost all target points. In this case, the ergodic invariant measure has a positive density. It is shown that the set of accessible states in discrete time coincides with the set of accessible states in continuous time up to a set of Lebesgue measure zero.

# The Inverse Mellin Transform Method and the Asymptotic Contour of Stationary Phase

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For the inverse Mellin transform, an integration path is proposed, based on the asymptotic of the stationary phase contour, which greatly accelerates the integral computation. The inverse Mellin transform method is widely used in the quantum chromodynamics analysis (QCD) of data for constructing the  $x$ -dependence of structure functions on the basis of their moments. The proposed approach for choice of contours makes it possible to significantly accelerate calculations in comparison with the standard linear paths of integration [1, 2]. This is especially important because of the increasing number of experimental points (more than a thousand) for the QCD-analysis of data on the polarized and unpolarized deep inelastic scattering of leptons on nuclei and the hadron production multiplicities. The proposed approach favorably differs from the approach based on the path of integration associated with the saddle point  $c_0$  of the integrand in the inverse Mellin integral [3]. The integral over such a contour begins to diverge with a significant removal from  $c_0$  and, as a rule, does not allow to restore the structure functions with an accuracy of eight to ten digits or more. The asymptotic contour of a stationary phase is applicable to calculation of a wide class of the Mellin-Barnes integrals arising in the quantum theory [4] and in practical application [5] and allows, if necessary calculate integrals them with an accuracy of ten digits or more.

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# On One Approximation of a Solution of the Stochastic Differential Equation with a Drift

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The stochastic differential equation of the following form

$$X_t = X_0 + \int_0^t \alpha(X_s, s) ds + \int_0^t \beta(X_{s-}, s) dW_s,$$

has been considered, where  $X_0 \in \mathbb{R}$ ,  $W_t$  is a Wiener process,  $t \in [0, 1]$ . Suppose that functions  $\alpha$  and  $\beta$  satisfy to conditions of the strong solution existence.

The main goal of the research is constituted to develop the method of an approximate calculation of the mathematical expectation  $\mathbb{E}[F(X_t)]$ , where  $F$  is a some functional satisfying to Lipshitz condition. The proposed method is based to be applying of approximation

$$\begin{aligned} \tilde{X}_t = X_0 + \int_0^t \alpha(X_0 + \alpha(a, t/2)s + \beta(a, t/2)W_s, s) ds + \\ \int_0^t \beta(X_0 + \alpha(a, t/2)s + \beta(a, t/2)W_{s-}, s) dW_s, \end{aligned}$$

where calibration parameter  $a$  is constituted to be either a number or a random value. The calibration parameter can calculated to be approximately (e.g. applying the formula proposed in [1]). In order to approximately calculate  $\mathbb{E}[F(X_t)]$  is used to be formulae from [2] applying to  $\tilde{X}_t$

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# DNA-RNA Transcription — Resonance Mode and Demodulated Standing Solitary Wave

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This work relies on the well-known helicoidal Peyrard-Bishop model (HPB) of DNA dynamics [1]. According to the model localized modulated solitary wave, usually called as breather, may move along the DNA chain. This wave consists of two components, an envelope and a carrier wave. They are represented by the secas hyperbolic and the cosine function, respectively.

The model can explain local opening of DNA using the idea of a resonance mode or extremely high amplitude mode [2]. This mode is relevant at segments where DNA chain is surrounded by RNA polymerase molecules, i.e. at the segments where DNA-RNA transcription occurs.

Here we study the resonance mode introducing the following two new ideas:

1. At these segments demodulation of the soliton occurs. Hence, we assume that the wave number and the frequency, existing in the cosine function, are zero.
2. We assume that the solitary wave becomes a standing one, which means that its speed, during the transcription, is zero.

It is explained why these ideas should be biologically convenient. Also, it is shown that this demodulated standing soliton, or DSS mode, is possible within the used HPB model.

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# Energy Levels of Electron in Circular Quantum Dot in the Presence of Spin-Orbit Interactions and Magnetic Field

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The energy levels are obtained within the framework of perturbation theory for an electron in the semiconductor circular quantum dot in the presence of the Rashba and Dresselhaus spin-orbit interactions of unequal strengths and the external uniform constant magnetic field. Confinement is simulated by the realistic potential well of finite depth.

## New Nonperturbative Phenomena of Quantum Chromodynamics

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We discuss the existence of new phenomena and properties of nonperturbative evolution of colour quarks, gluons and other states in stochastic vacuum of quantum chromodynamics, colour dissipation and confinement; instability of movement of colour particles in confinement region; appearance of squeezed and entangled states of strongly interacting particles; correlation properties of strong instantons decays; chaos assisted instanton tunneling; quark-gluon plasma properties description in terms of Hagedorn bootstrap statistical mode.

# Estimates for Solution of the Crocco Typical Boundary Problem

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**Keywords:** Crocco typical boundary problem, Mascheroni constant

For Crocco typical boundary problem there exists a solutions approximation sequential process by entire functions. One is formulated to be follows

$$2\varphi \frac{d^2\varphi}{du^2} + u = 0, \quad D(\varphi) = \{u: 0 \leq u_0 < u \leq 1\}, \quad \left. \frac{d\varphi}{du} \right|_{u_0} = 0.$$

The solutions properties are set up to be the following statements.

1° Let  $u_0 \geq 0$ . Then there exist two branches of the solution of the typical boundary problem (C), positive function  $\varphi^+$  and negative function  $\varphi^-$ , such that

$$\varphi = \varphi^+ \geq 0: \exists a > 0 \Rightarrow \varphi^+(0) - a = 0, a \geq \varphi(u) \geq 0, \frac{d\varphi^+}{du} > 0, \frac{d^2\varphi^+}{du^2} > 0,$$

$$\varphi = \varphi^- \leq 0: \exists a < 0 \Rightarrow \varphi^-(0) - a = 0, a \leq \varphi(u) \leq 0, \frac{d\varphi^-}{du} > 0, \frac{d^2\varphi^-}{du^2} > 0.$$

Therefore  $\varphi: (u_0, 1) \rightarrow (0, a)$  and the map is injective

$$\varphi(u') = \varphi(u'') \Leftrightarrow u' = u''.$$

2° Crocco typical boundary problem is equivalent to an integral equation for  $\varphi(u)$

$$\varphi(u) = \frac{1}{2} \int_u^1 d\nu \int_{u_0}^\nu \frac{tdt}{\varphi(t)};$$

3° It is faithful equality

$$\int_1^{u_0} \left( \frac{d\varphi}{du} \right)^2 = \frac{1 - u_0^2}{4},$$



therefore  $\varphi \in W_2^{(1)}(u_0, 1)$ ;

along to characteristics of the Crocco equation the condition

$$F(\varphi) = \frac{1}{2} \int_{u_0}^1 \left( \left( \frac{d\varphi}{du} \right)^2 + u \log \frac{a}{\varphi} \right) du \rightarrow \inf \geq 0$$

is faithful; it is really the Crocco equation gives a necessary condition of minimum for  $F(\varphi) \geq 0$ .

4° Let  $u_0 = 0$ . Then  $\varphi_\theta^2(u) = 1/6(1 - \theta^2)(1 - u^3)$  where  $\theta$  be a proper fraction. The arithmetic average on  $\theta$  value is expressed to be equality

$$\varphi^2(u) = 1/9(1 - u^3), \quad \varphi(0) = 1/3$$

(the exact value is defined to be  $\varphi(0) = 0.33206 \dots$ ).

5° In general, the numerical sequence

$$a_r \stackrel{\text{def}}{=} \|\varphi(0)\|_r \stackrel{\text{def}}{=} \frac{1}{\sqrt{6}} \left( \int_0^1 (1 - \theta^2) d\theta \right)^{1/r}$$

is limited for all  $r > 0$  and one the sequence is increased to be increase at index  $r$  at that

$$a_0 = \frac{\exp(\psi(1)/2 - \psi(3/2)/2)}{\sqrt{6}} = \frac{\exp(-c/2 - 0.0182)}{\sqrt{6}} < a_\infty = 1/\sqrt{6}$$

where  $c = 0.5772$  being Mascheroni constant.

# Onset of Wada Basins From the Cycles

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**Keywords:** dissipative dynamic system, saddle fixed point, Wada basins, top of umbrella

*Wada basins* are called to be the a simply connected regions on the plane with compact closure having common boundary. It is clear that can exist at least two Wada basins and there exists (*Wada*) *ocean* being the interior of the addition for all Wada basins union and it having common boundary with all Wada basins.

There considers onset of two Wada basins and (Wada) ocean from two cycles  $C_1$  and  $C_2$  are invariant with respect to the dissipative action  $\psi_k \in \text{Diff}(\mathbb{E}^2)$  with evolutionary parameter  $\omega$ . In order to track evolution of the cycles into Wada basins the system of nested disks  $D_n$  containing saddle fixed point  $p$ . The disks diameters are formed to be decreasing sequence  $1/n$ ,  $n \in \mathbb{N}$ . Parameter value  $\omega = \omega_0$  at  $n = 1$  is defined to be follows condition  $W^s(p) \cap D_1$  is connected. . . Such away parameter value  $\omega = \omega_{\nu-1}$  at  $n = \nu$  is defined to be follows condition  $W^s(p) \cap D_\nu$  is connected but previous intersection  $W^s(p) \cap D_{\nu-1}$  is not connected. Therefore there are form two sequences  $\text{Seq}(\omega, \nu - 1)$  being real number and  $\text{Seq}(\nu, n)$  being integer. Moreover it is proved to be following

**Theorem 1.** *If dissipative action  $\psi_k \in \text{Diff}(\mathbb{E}^2)$  possesses Wada property then saddle fixed point be a top of umbrella, and is not accessible.*

# Mathematical Modeling of Processes with High Energy Density

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**Keywords:** mathematical modeling, high energy density, electrical discharges, nanofabrication of materials

Processes with high energy density are widely used in various fields of science and technology. For example, a new impetus for the development of electro physics is associated with using different types of electrical discharges in biology and medicine. These applications are based on their energetic and non-toxic factors affecting the medium on a cellular level. High efficiency energy conversion, introduced into the plasma discharge to the energy of fluid motion, provides various bio chemical applications of such physical processes. Also there is a growing interest in the nanofabrication of materials and their applications in various fields of life and technology, such as electronics, energy generation, health care and storage. A great deal of progress in this field has relied on the use of lasers. Production of nanoparticles can be done in several ways; one of them is laser ablation. For the study of such processes, mathematical models have been developed. These models contain the equations of magnetic hydrodynamics and the evolution of electric and magnetic fields. To complete the system of equations, it is necessary to add the equations of state, the selection of which represents a separate and complex problem. An equally important problem is the determination of the thermodynamic parameters entering into the equations in the form of variable coefficients. To verify the adequacy of the models, the calculated data are compared with the experimental data. An important result is the confirmation that it is necessary to use different equations of state in various areas of the process parameters, such as temperature, pressure, and others. Most accurately, the state of the substance can be determined by invoking the laws of quantum mechanics and statistical physics.

# Dissipative Solitons in Biopolymers

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Thermal fluctuations play a critical role in functioning of biomolecules. At normal temperatures most of collective vibrational modes are overdamped. However, relatively strong nonlinearity of such a system in conditions of energy supply may give a rise to formation of dissipative structures.

Here we consider several important types of dissipative structures in cytoskeleton proteins, subjected to action of electromagnetic or chemical pumping. As an example of low-frequency processes (radiofrequency band), a coupled propagation of mechanical vibrations of terminal carboxyl groups (C-termini) and the current of electrolyte ions in cellular microtubules is considered. It is shown that metastable states ensure the existence of wave fronts or autowave signals for which analytical traveling wave solutions were obtained. Also, the conditions for generating oscillating signals are determined.

As an example of high-frequency processes in microtubules (THz frequency band), the dynamics of tunnel transitions in tubulin dimers is considered. It is determined that an alternating external electromagnetic field with a resonant frequency is capable to support localized oscillatory excitation (dissipative soliton) in the microtubule chain. The description of their dynamics reduces to a complex equation of the Ginzburg-Landau type.

# Quantum Chemical Computations of Topological Phases in DNA Molecule and the Triplet Nature of the Genetic Code

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**Keywords:** quantum chemical computations, nonabelian quantum topological group, topological phase, DNA, nucleotide, genetic code

The research aims at elucidating the nature of the genetic code – why the genetic code is triplet and what information is stored in a three-letter nucleotide sequence. As the encoded information is stored in DNA, this molecule (its full turn, to be specific) serves the object of the study. The research rests on quantum chemical computations (ArgonneLab-2017 program code) of quantum topological phases with fractional statistics. The carriers of quantum topological phases (anyons) in DNA are nucleotides, which triplets form the unique nonabelian quantum topological groups – separate codes with the genetic information built in. The nature of this information, i.e. what is encoded in a sequence of three nucleotides, is still unclear. The research is absolutely new with an output into constructing a three-level quantum computing.

# Can Nonlinearity be Beneficial for Supporting Bloch Oscillations and Dynamic Localization?

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Bloch oscillations is one of the most fundamental physical phenomena which is intensively discussed in the literature over the last eight decades, since it was predicted in the seminal works by Bloch and Zener in the theory of electrons in lattice potentials [1, 2]. In the early 1990s Bloch oscillations (BOs) were first observed experimentally in electrically-biased semiconductor superlattices using optical interband excitation with femtosecond laser pulses [3]. A few years later, also for atoms in optical lattices [4] and for coupled waveguides [5] BOs have been realized. By its nature BO is a linear phenomenon and it is common belief that nonlinearity plays a destructive role which makes it impossible to observe BOs at long times (or propagation distances, depending on the particular physical context) even with without dephasing processes. This was first reported in [6] and later on confirmed experimentally in optics using arrays of Kerr-type waveguides [7] and furthermore in Bose-Einstein condensates (BECs) loaded in optical lattices [8], where only a few oscillations were detected.

However, balance between the linear and nonlinear effects can be achieved in systems that contain an additional continuous dimension besides the one corresponding to the direction of the linear gradient. This balance results in very stable oscillatory motion of discrete-continuous hybrid BO-breather nonlinear waves [9, 10]. Such oscillations can be observed even for moderate nonlinearities and large enough values of linear potential, when the band-gap picture of the underlying linear lattice is not applicable anymore. We will demonstrate that there exists an optimal relation between nonlinearity and linear gradient strengths allowing for extremely long lived BOs. Also

we will demonstrate a constructive role of nonlinearity in another famous phenomenon related to BOs-the dynamic localization.

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# Encryption of Images with Applying Chaotic Maps and Parallel Computing

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One most promising trends in modern cryptography is constituted to be elaboration of the encryption algorithms based on the dynamic (deterministic) chaos properties. There presented an image encryption algorithm based on the chaotic dynamics principles and being optimized for parallel computing. A raster images compression is used to be decrease information involved in the encryption process by means of the discrete wavelet transformation. The resultant image is divided to be 4 subranges and one of them contains a reduction of the initial image. This range is encrypted separately to using the algorithm of the confusion-diffusion. The confusion, diffusion stages and the key generator are realized to be chaotic mapping. The tent mapping is applied to be confusion and diffusion

$$f(x_n) = \begin{cases} \mu x_n & \text{for } x > 1/2 \\ \mu(1 - x_n) & \text{for } x \leq 1/2 \end{cases}$$

In order to compress an image it is applied embedded coding of the refined wavelet coefficients by the HBCT algorithm (Hardware Block Cluster Tree). The refined coefficients are encrypted to using the block-encrypted algorithm based on reversible cellular automatons. The raster image compression and the employed parallel computing is used to increase performance of the encryption algorithm. On the stages of wavelet transformations if the main subrange containing a reduction of the initial image has been encrypted then operations are performed within the limits of the blocks with fixed dimensions enabling one to perform the computations in parallel. Unspecialized computations with a graphic processor is used to be parallel computing.

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# A New Cryptosystem Based on Fractional Order Time-Delay Chaotic Systems and Wavelet

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We propose a new cryptosystem based on fractional order time-delay chaotic systems and wavelet. We focus on the security of images transmissions. The general principle of this approach is to drown the original image into the chaos before inserting it in the approximation of another image by discrete wavelet transform (DWT). At the receiver level, we use a nonlinear state observer design for establishing the synchronization between the transmitter and the receiver, also for recovering the transmitted image. The numerical simulations are provided to verify the effectiveness and feasibility of the developed method.

# A New Contribution for Predictive Synchronization of Fractional Order Time-Delay Chaotic Systems with Application for Secure Communication

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We present a new contribution for synchronization of fractional order time-delay chaotic systems with an application for secure communication. The feedbacks predictive control is used to control this chaotic system. generally , the gain of the predictive control used in the literature is taken as a constant included in an interval, however, in this work, this gain is taken as a matrix, and Linear matrix inequality is using to calculate this gain. This paper also extends the above idea to secure communications, where, the transmitted message is injected in the dynamics of the driver chaotic system. after synchronization with the proposed approach, we use the chaotic demodulation to recover the message. The obtained results show clearly the effectiveness of the proposed approach.

# Tenebrarum Dynamicus et Philosophia a Oeconomia: Stable Generalized Simple Commodity Production

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**Keywords:** a (generalized) simple commodity production, chaotic dynamics, dissipative dynamic system, saddle and antisaddle fixed point, Euler characteristics, stationary point, homoclinic point, nonwandering set, indecomposable continuum (atom), lakes of Wada (Wada basins), Birchoff's curve

*Simple commodity production* is called to be production based on private/personal ownership of the manufacturer and his personal work. Engels is coined to be the term in order to describe productive activities under the conditions of what Marx had called the "*simple exchange*" of commodities, where independent producers trade their own products (see Frederick Engels, *Afterword* to Vol. 3 of *Das Kapital*).

Generalized simple commodity production has been considered from the chaotic dynamics viewpoint. A Production is observed to be neighborhood saddle (or inverse saddle) fixed point of the dynamic system action in the *resources space*.

One demonstrates the homoclinic point, Birkhoff's curve and Wada basins (lakes of Wada) existence: there exist both two sources (unstable antisaddle stationary points) of human and natural resources and the generalized simple commodity production being both the source and drain (saddle or stationary point with many separatrices). Another words the stable generalized simple commodity production dynamic system action possesses the Wada property.

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