Petrovič's theory of elements of mathematical phenomenology and phenomenological mapping applied to system nonlinear dynamics

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Summary. Using Mihailo Petrovič's theory of mathematical phenomenology elements, phenomenological mapping in non-linear dynamics, linear and non-linear vibrations, signals, main and parametric resonance and dynamical absorptions, resonant jumps, trigger of coupled singularities, trigger of one side singularities, in global and local models of system dynamics – abstractions of different real system local and global dynamics are identified and presented.

Preface

Mihailo Petrović Alas, mathematician, presented in two books [1,2] a theory containing *elements of mathematical phenomenology* and *phenomenological mapping*. Alas' theory, between other, defines two types of analogy: qualitative and mathematical analogy. In the time of computer and software tool expansion, Roger Penrose and James Glaick had similar ideas that were later applied in graphical –computer techniques.
Phenomenological mapping of phenomenon and models enables multiple system dynamics models of disparate nature to be described by a single mathematical model: for example an electric chain model and a model of a mechanical chain with the same degree of dynamics freedom.

Introduction

According to our knowledge, there are only several studies available in the literature discussing phenomenological mapping: in mathematics [3-5], or as a part of string theory and its applications [6]. Rašković [7] gave series of examples of electro-mechanical analogous vibration systems mathematically described and solved. Hedrih (Stevanović) [8] showed analogy between vector models of stress state, strain state and state of the body mass inertia moments. Ideas of mathematical phenomenology and phenomenological mapping, from listed references, are used for investigating dynamics, and vibration phenomena of resonance and dynamical absorption for solving series of research problems of dynamics of various kinds of chain systems [9]; In classical books chain system dynamics are usually described for systems for up to three degree of freedom. Investigating resonance and dynamical absorption in chain dynamics in the systems with more then three degrees of freedom using phenomenological mapping are important not only for mechanical signal processing, but also for electrical signal processing and signal filtering, for processing biodynamical signals in life systems (DNA double helix chains).
This approach makes possible an integration of contemporary knowledge obtained in various areas of sciences and reduction of qualitative models or corresponding mathematical models to a minimum needed to describe dynamics of real systems with various physical properties. We can describe both the mechanical signal through mechanical chain and the electrical one, with same equations although they describe different physical phenomena. In both systems a set of own circular frequencies, resonance, dynamical absorption and signal filtering are in question regardless whether it is mechanical motion or an electrical signal.
Phenomenon defined as dynamical absorption appears only in conservative systems with two and more degrees of freedom in the forced regimes when a single frequency force is applied at one mass particle, and this mass particle is in the forced rest, and other mass particle in the system vibrate in forced regimes. According to our knowledge, this phenomenon is not described in enough detail and investigated in the literature for the system with more then three degrees of freedom, as well as numerous resonances.
The aim of our research was to investigate classical models of free and forced dynamics in linear and nonlinear chains in with finite number of degrees of freedom, and with corresponding numerical analysis of the system with multi degrees of freedom; to analyse the existence of dynamical absorption at amplitude of mass particles displacement in forced regime of the system vibrations.

Methods

By using a mathematical description of a chain mechanical system with finite number of mass particles coupled by linear as well as non-linear elastic springs or fractional order elements with finite number of degrees of freedom expressed by corresponding generalized independent coordinates and corresponding analysis of solutions for free and forced vibrations series of multi-frequency regimes, resonant states and dynamical absorption states are identified. Also, by using qualitative and mathematical analogy and phenomenological mapping analysis of series of dynamics of other chainlike models is made.

We use phenomenological mapping to explain dynamics in systems with multiple deformable bodies (beams, plates, membranes or belts) through resonance and dynamical absorptions in the system and transfer of mechanical energies between bodies. Elements of mathematical phenomenology- qualitative and mathematical analogy are used to make a transfer for all results obtained for forced dynamics of a mechanical chain system to other chain systems, like torsion chain system (gear chain machines), multi-pendulum chain system, electrical circle chain system forced vibrations, DNA helix chain system dynamics.
Amplitude-frequency graphs for homogeneous and non- homogeneous chain systems are presented for a system with multiple degrees of freedom. Expressions for generalized coordinates of chain non-homogeneous system in resonance regimes for general case are derived. Three theorems are defined and proven.

Main results

A number of models of the real mechanical system nonlinear dynamics approximations with ideal and nonideal constraints and in present friction between moving elements are presented. These models, at the same time, present corresponding models with qualitative and mathematical analogous nonlinear dynamics. Two models on nonlinear dynamics with numerous nonlinear phenomena are a heavy mass particle nonlinear dynamics along rotating circle about a fixed vertical as well as skew positioned axis, with constant angular velocity and with different case of contact between mass particle and circle line (ideal, non ideal with sliding frictions Coulombs type). These two models of complex nonlinear dynamics are suitable to point out numerous nonlinear properties such as: trigger of coupled singularities, one side singularities, homoclinic orbits in the form of number “eight”, local and global dynamics properties, relative equilibrium positions and bifurcation of the equilibrium positions, main resonance and parametric resonance and other. Linearizations and linear and nonlinear approximations of corresponding differential equations about stationary points (stationary regimes) are obtained. Analysis of stability and instability of system nonlinear local and global dynamics around stationary points (stationary regimes) as well as around system relative equilibrium positions is presented. Also, reasons for investigating the character of local properties of linearlike as well as nonlinear dynamics around singular points are presented.

Conclusions

Petrović’s theory of elements of mathematical phenomenology is a suitable tool for integration of some knowledge in different areas of nonlinear dynamics and useful for reductions of models of nonlinear dynamics containing complexity of nonlinear phenomena – regular, chaotic and stochastic regimes with founds knowledge to explain nonlinear dynamics of different physical, chemical or biological natures as well as social dynamics properties. Using some of models, that includes linearization or linear and nonlinear approximation around stationary states it is possible to investigate local dynamics contained in global nonlinear dynamics as a good way for transfer knowledge in one area of science into other.

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References

[1] Petrović M (1911) Elementi matematičke fenomenologije (Elements of mathematical phenomenology), Srpska kraljevska akademija, Beograd, pp. 89. http://elibrary.matf.bg.ac.rs/handle/123456789/476?locale-attribute=sr
[2] Petrović M (1933) Fenomenološko preslikavanje (Phenomenological mapping), Srpska kraljevska akademija, Beograd, pp. 33.
[3] Frazer RW (1961) Applications of Conformal Mapping to the Phenomenological Representation of Scattering Amplitudes. *Physical review* 123 (6): 2180-2182.
[4] Freudenthal H (1986) Didactical Phenomenology of Mathematical Structures. Springer .
[5] Pettifor D G (1986) The structures of binary compounds. I. Phenomenological structure maps *Journal of Physics. C: Solid State Physics* 19: 285 doi:10.1088/0022-3719/19/3/002
[6] Kelebanov RI and Maldacena MJ (2009) Solving quantum field theories via curved spacetimes. *Physics Today* 68: 28-33.
[7] Rašković D (1974) Analitička mehanika (Analytical Mechanics), Mašinski fakultet Kragujevac
[8] Hedrih (Stevanović) K (1991) Analogy between models of stress state, strain state and state of the body mass inertia moments. *Facta Universitatis, Series Mechanics, Automatic Control and Robotics* 1(1): 105-120.
[9] Filipovic D and Schroder D (1999) Vibration Absorption With Linear Active Resonators: Continuous and Discrete Time Design and Analysis. *Journal of Vibration and Control.* 5(5): 685-708.