Elements of mathematical phenomenology and qualitative /mathematical analogies on the basis of generalized Lissajous curves

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Summary. Generalized Lisajous curves are defined as results in sum of orthogonal multi-frequency vibrations. Many different and various forms of generalized Lissajous curves with singular forms and attractors appear as possible graphical presentations of many different dynamics. Elements of mathematical phenomenology and qualitative /mathematical analogies on the basis of generalized Lisajus curves open numerous possibilities for investigation, identifications and analysis of different nonlinear dynamical regimes and local phenomena as it is: periodic, regular or chaotic dynamics, bifurcation of equilibrium positions, synchronization, asynchronization, double periodic dynamics in nonlinear dynamical systems.

Preface

Jules Antoine Lissajous, (Born: 4 March 1822 in Versailles, Died: 24 June 1880 in Plombières, France) was a French mathematician. Among other innovations, Lissajous invented the Lissajous apparatus, a device which creates the figures, after whom Lissajous figures are named. In it a beam of light is bounced off a mirror attached to a vibrating tuning fork, and then reflected off a second mirror attached to a perpendicularly orientated vibrating tuning fork (usually of a different pitch, creating a specific harmonic interval), onto a wall, resulting in a Lissajous figure. This led to the invention of other apparatus such as the harmonograph.
Mihailo Petrović, important Serbian mathematician, finished Sorbonne University (and his Professors were Ch. Hermite, É. Picard, P. Painlevé and also H. Poincaré, Darbox, Apell, Tannery, Boussinesq, Koenigs, Lippmann), presented in two books (Petrović, 1911, 1933) [6.7] a theory containing elements of mathematical phenomenology and phenomenological mapping. But, both publications were published in Serbian and only a small number of his contemporaries (between them important world scientist Milutin Milanković, author of importunate Canon of Sun insulation), were able to read and understand this theory. Petrovic's theory, between other, defines two types of analogy: qualitative and mathematical analogy. In the time of computer and software tool expansion, Roger Penrose (1989) [8] and James Glaick (1987) [1] had similar ideas that were later applied in graphical –computer techniques.

Generalized Lissajous coves and synchronization in the multi body systems

Series of Lissajous curves as coupled orthogonal component single-frequency synchronous or asynchronous vibration with corresponding propertie are known from literatures. New series of the generalized Lissajous curves are obtained by software MathCad as results of the coupled orthogonal, both, component multi-frequency oscillations (see Figure 1). A method based on attractors of asynchronization/synchronization of the component multi frequency oscillations of the subsystems of hybrid system is presented in Reference [2]. Generalized Lissajous curves are used as attractors of asynchronization/synchronization of the component oscillations which are coupled as that these oscillations are orthogonal (see Figure 2 and Reference [10]). By changing some parameters of the coupled oscillators synchronization and by use current software tools as it is MathCad, the visualization of the transformation of the generalized Lissajous curve, up to its degeneration into part of straight line, can be obtain as results of the orthogonal coupling of oscillatory multi-frequency signals. If this degeneration is not possible, then these oscillators are not possible to synchronized and corresponding parameter is not parameter of synchronization. If as results of the change of some parameters of the coupled oscillators synchronization is transformation of the generalized Lissajous curve into one unique line then it is possible to obtain system parameters of the attractor of partially synchronization or asynchronization of the coupled oscillators

Qualitative /mathematical analogies on the basis of generalized Lissajous curves

Generalized Lissajous curve can present phase trajectories in phase plane, trajectories of the motions and also other graphical presentation of the dynamics regimes of physically different real system model dynamics. By these curves is possible to investigate singular points, trigger of coupled singularities, character of structural stability of the nonlinear local and global dynamics. In our investigation there are numerous results in area of multi-body dynamics, physic-chemical reactions, biodynamical systems from which is possible to identify elements of qualitative as well as mathematical analogy on the basis of use generalized Lissajous curves for analysis local and global nonlinear dynamics properties (see for examples Figure 1, 2 and 3 amd References [3], [4], [5] and [9]).

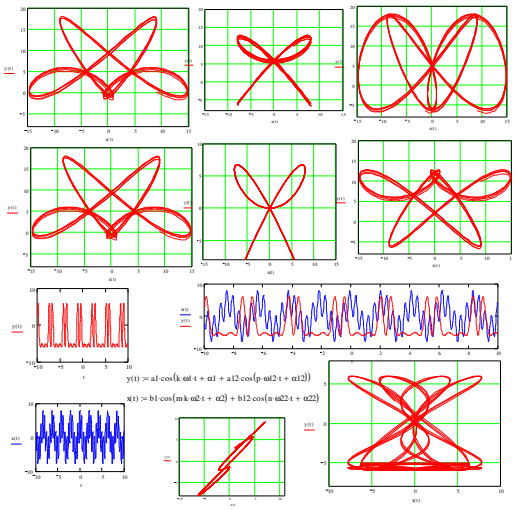

Figure1. Generalized Lissajous curves in results sum of two orthogonal both twofrequency oscullations [2].

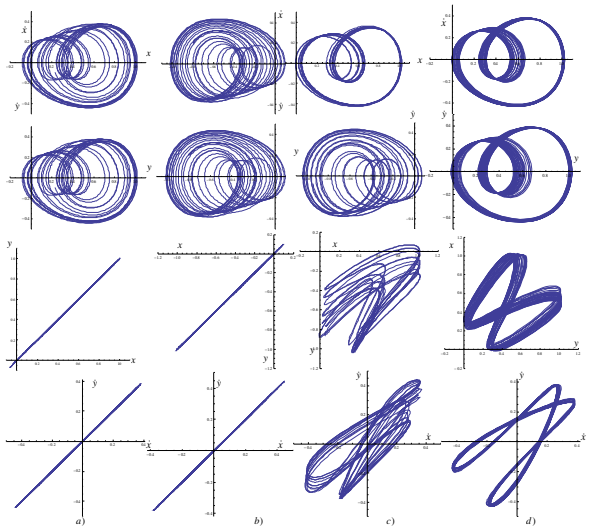

Figure 2. Transformation of the phase trajectoriy portraits by change of two parameters and initial conditions of two plate coupled by a viscoelastic discrete continuum layer [10]

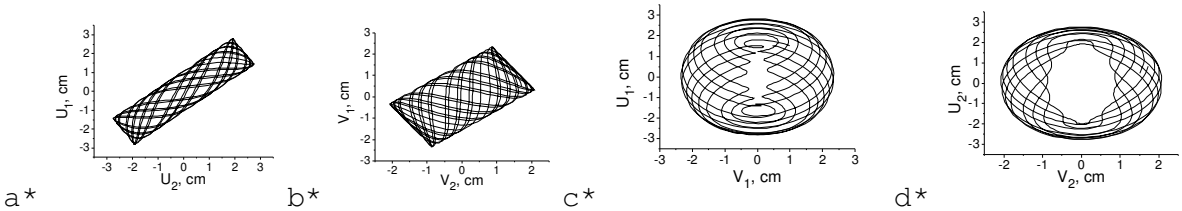

Figure 3. Phase trajectory portraits of two-mass particle displacements and velocities. In a* and b* phase trajectories $(u_1(t),u_2(t))$ and $(v_1(t)=\dot{u}_1(t),v_2(t)=\dot{u}_2(t))$ and in c* and d* phase trajectories $(u_1(t),v_1(t)=\dot{u}_1(t))$ and $(u_2(t),v_2(t)=\dot{u}_2(t))$.

Conclusions

Elements of mathematical phenomenology and qualitative /mathematical analogies on the basis of generalized Lisajus curves open numerous possibilities for investigation, identifications and analysis of different nonlinear dynamical regimes and local phenomena as it is: periodic, regular or chaotic dynamics, bifurcation of equilibrium positions, synchronization, asynchronization, double periodic dynamics in nonlinear dynamical systems.

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