Using GIS in Dynamic Rellocation of Emergency Ambulance Vehicles

Čedomir Vasić[1] and Bratislav Predić[2]

[1] Dona Farm, Blvd. Dr. Zorana Djindjića 40a, 18000 Niš, Serbia
[2]University of Niš, Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Niš, Serbia

Abstract— Data about routes of ambulance vehicles in period of extensive number of days, extracted from database connected to GIS, are used in calculations of coordinates related to location where these vehicles should be parked during „waiting for call“ time. Mathematic problem known as „p-median“ is resolved by apropriate algorithm implemanted in three steps. Defined optimisation method is tested on site

Keywords— GIS, Emergency Ambulance Service, rellocation, p-median

I. Introduction

Problem of location-allocation of resources was resolved on different ways many times in many different fields of researching, not only these days but also in last century. Fundamental advance is made on first place with developing of spatial model based on GIS (Geographical Information System) and growth of computional power available to researchers. Up to date units for tracking movement of vehicles using GPS (Global Positioning System) and software used in GIS, allow dynamic analizing and optimal decision making process when we talk about location-allocation problem of different purpose vehicles. Researchers keep in focus of interest problems of planing services in public sector. Emergency medical help and ambulance services are specialy targeted fields, according to human nature of service users.

In processes of planing services in Emergency sector on deterministic way on the begining you must have answers on couple different questions: how many vehicles service had to purchase, how many vechicles had to be available in any moment, where to build garages etc. When service is already estabilished, you had to provide mechanism of decision making and answer offering, after every single call: which vehicle to use (which vechicle is alocated) and location of parking place for vehicle in status of „waiting for next call“ (rellocation of vehicle). Rellocation problem is simplified as we need to find location for best space covering and in same time location which provide shortest time of response.

If we make asumption that we have no influence on technical level of equipment used in Emergency services and make asumption that resolving of that problem is out of scope of this paper, we will see that any improvement in quality services is possible only on one of these two ways: better treining of personel or shortest travel time of ambulance in attempt to reach the site. It is obvious if all vechicles are parked in one central garage and all of them start from same position it is not optimal way if aim is to have shortest average time of response and reach patient as soon as it posible. History of routes of ambulances in defined period of time in recent period and spatial location of served calls, defined thrue data from GIS, are excellent base and start position to resolve rellocation problem. If we can perform data analize and recalculation of optimal location in any time, we achived goal and we posses ideal tool for dinamic rellocation of available recourses. Although goal to find the most quickest route to destination suffer from many complex impacts and many different conditions influence in decision making process it is more than obvious that main influence have Euclidian distance. Trafic jams, rush hours, road reconstruction, weather conditions, time of the year and other conditions need to be involved in dynamic allocation modelling. In our case we made assumption that ambulance vehicle can be parked and wait for call in any location from which call for service is lounched in last 30 days. We assumed that main impact on response time have distance between two points, and we allow that other relevant issues can be used in account in future. Goal was to create ambulance rellocation tool using available data and find coordinates of parking place which will provide minimized average response time. As main benefit we get a higher level of quality of this very inportant public service.

II. P-median Problem

The problem how to locate p-facilities (called medians), so as to minimize the sum of the distances from each demand point to its nearest facility, is clearly defined in early sixties of last century as a p-median problem. The resolving of this problem is classified as Non-deterministic Polynomial-time (NP) hard problem. For even moderate values of demand points n and facilities p, the number of possible solutions can be very large and it is defined by:

$$\binom{n}{p} = \frac{n!}{p!(n-p)!} . \quad (1)$$

For instance, if n=1000 and p=10, the total amount of possible solutions is $2.63 \cdot 10^{23}$. In this paper, p-median problem is modelled as the binary integer programming problem, explained as follows. Find minimum of:

$$\sum_{i \in I} \sum_{j \in J} \omega_j d_{ij} y_{ij} \quad (2)$$

subject to constraints:

$$\sum_{i \in I} x_i = p \quad (3)$$

$$\sum_{i \in I} y_{ij} = 1, \quad \forall j \notin J \quad (4)$$

$$y_{ij} - x_i \leq 0, \quad \forall i \in I, j \in J \ (5)$$

$$x_i \in \{0,1\}, \quad \forall i \in I \quad (6)$$

$$y_{ij} \in \{0,1\}, \quad \forall i \in I, j \in J \quad (7)$$

where ω_j is weighting factor of location *j*, d_{ij} is an Euclidian distance between *i* location for vehicle parking and *j* demand for service. The constraint (3) defines that *p* is a total number of vehicles which we need to relocate, (4) tells that one demand can be served only by one vehicle, (5) suppress possibility to serve demand from location without a vehicle, (6) shows that on one parking place can be only one whole vehicle (vehicle can not be divided on several locations) and in the end (7) ensures that one demand can be serviced with only one vehicle (not with two halves of two different vehicles). Some of the constraints can be removed or avoided and that is the relaxation of problem. Some additional constraints can be introduced. For example, it is possible to define a maximum value for d_{ij}, establishing a maximum allowed call response time in that way, or to determine a maximum allowed number of calls serviced by one vehicle (the capacity constraints).

III. Previous works

P-median problem is for first time defined in works of Hakimi [1]. For the first time continuous surface of space is devided in discrete network and Hakimi assumed that medians can be placed only in graph vertices. He give mathematicaly proof that there is at least one optimum solution of problem [2]. He proposed simple enumeration procedure named „direct enumeration“ for calculation of one or more medians.

Direct numeric enumeration and heuristics were the earliest techniques proposed. ReVelle and Swain [3] provided the first linear programming formulation of problem and involved integer variables in numeric resolving. These work is unavoidable basis for all later proposed methods.

For large amount of numbers *n* and *p* direct enumeration is not acceptable solution, and methods which provide near exact solution are introduced. Mathods named heuristic are developed as two phase algorithms. First phase is always dedicated to efforts to find starting set of p-medians, and second phase is made from iterations dedicated for solution improveing. Final solution is very close to optimal, or sometimes it is optimal solution. There were three primary early heuristics: Greedy, Alternate and Vertex Substitution.

Greedy method is intoduced in praxis due to works of Kuehn and Hamburger [4] based for the warehouse location problem. Greedy heuristic initially chooses *n* locations that maximize the cost savings of replaceing these *n* locations with warehouses. It than considers each of these locations individually and calculates the total distribution cost. Any location that does not reduce the total cost is eliminated from further consideration. The location that give the minimum cost is assigned a warehouse. This process is repeated until all elements of the original list have been eliminated or assigned as a warehouse.

Alternate heuristic proposed by Maranzana [5] define finding of p-median that, after each iteration, yields a collection of p-vertices that is guaranted same or better from previous set. Algorithm begins with an arbitrary set of p-vertices. It is proved that algorythm lineary converge to an local optimum, but arbitrary choosen starting set can lead process to region from which optimum can`t be reached.

Teitz and Bart [6] defined method known as „vertex substitution“ or interchange heuristic. This solution is commonly used nowdays as „one-opt“ procedure. Starting set of medians is derived thrue exact calculation of „first median“. This aproach and direct calculation of starting medians is acceptable but inconvinient if networks with many nodes are used. In that case, it is more convinient to reach the starting set of medians with calculating less computational demanding way. In second phase of algorythm all nodes are rearranged in either of p-subsets according of process of neighborhooding. On every subset procedure of „first median“ calculation is again aplied. If new candidate is better solution vertices are switched and whole process is repeated on the new solution. When all subsets have been checked, algorythm terminates with a local minimum solution.

Densham and Rushton [24] improved „vertex substitution“ method and atention give to creating an informed spatial search procedure that is more efficient and more effective than the original native spatial search procedure. This procedure is implemented in new method called Global Regional Interchange Algorythm (GRIA).

„Vertex substitution“ is in common life the most used procedure and Whitaker [25] and later Resende and Weneck [26] showed that there are lot of possibiliteis for further improvements.

If it is impossible to find starting set of medians by direct enumeration, methods known as Lagrange relaxation are introduced and iterations are implemented on several different ways: subgradient-relaxation or growing-optimisation, brunch and bound, double-incrementing or „surogat“ optimisation. Milestone work is algorithm of Narula, Ogibu and Samuelsson [22] which begins with „good“ initial value for the dual variable and moves in direction where subgradient is not zero. Process terminates when all components of subgradient are zero or when there is no duality gap.

Some of most succesfull methods based on Lagrange relaxation are proposed by Cornuejols, Fisher and Nemhauser [19], Hribar and Daskin [20], Khumawale [21] etc. The newest „Surrogate“ relaxation from Senne and Lorane [23] use new aproach to find range of multiplier values that improve the bounds of the usual relaxation technique. It is possible to generate approximative solutions at least as good as traditional relaxation technique while reducing computational effort for larger problems.

Last few years there are many papers explaining metaheuristic and approximation algorithms. The most common metaheuristics are „Variable Neighborhood Search“, „Genetic Algorithms“, „Tabu Search“, „Heuristic Concentration“, „Simulated Anneling“ and „Neural Networks“.

Hansen and Mladenović [7] developed method of „Variable Neighborhood Search“ (VNS) that involves a systematic change of neighborhood within a local search algorithm. The process involves exploring increasingly distant neighborhoods to avoid local minimum. Variations of these methods are „Parallel Variable Neighborhood Search“ (PVNS) from Garcia, Batista, Perez and Moreno-Vega [8], „Cooperative Parallel Variable Neighborhood Search“ (CPVNS) from Crainic, Gendreau, Hansen and Mladenović [9] etc.

Resende and Werneck [10] intoduces a randomized multistart iterative metaheuristics known as “Greedy Randomiyed Adaptive Search Procedure” (GRASP). Each iteration of this process applies algoritm followed by a local search procedure. A pool of best solutions of previous iterations is stored, and after each iteration, a new candidate solution is combined with stored solution in a process called “path-relinking”. Once the algorithm terminates, the stored solutions are combined with each other.

„Heuristic Concentration“ developed by Rosing and ReVelle [11] is two stage solution. In first stage method creates a concentration set that contains vertices with high probability of being facilities in the optimal solution. The second stage involves using the exact algorithm to solve a subproblem on this set. On examples they proved that concentracion is superior method in compare to vertex substitution in aim to get starting set of p-medians. If we randomly choose locations for starting set (as proposed by Maranzana) it is 20% probability to fall in loop which leave us far from optimal solution. In same time if we apply concentation, this risk is only 5%.

Rolland, Schilling and Current [12] defined metaheuristic called „Tabu Procedure“. This method involves tabu restrictions, aspiration criteria, deversification and strategic oscillation. For example, restrictions prevent the search from moving back to previous bad solutions. Deversification is used to escape from a local mimimum by restricting search to make same moves too often etc. Salhi [16] make further improvements in „Tabu Procedure“ involving a new aspiration criterias.

„Simulated Anneling“ is among metaheuristics derived in attempts to use principles spoted in nature as template for mathematics algorithm. Mathematical implementation of cooling structure and deterministic version of mean-field annealing is for the first time explained by Righini [17] and later advanced bu Chiyoshi and Galvao [18]. Among newest metaheuristics are „Genetic Algorithms“ proposed by Hosage i Goodchild [15], and extended in works of Alp, Erkurt and Drezner [13] and also Lorena and Furtado [14]. Genetic algorithms are problem resolveig methods inspiried by Darwin teory of natural evolution and to final set of medians we come thrue several generation of sets. These sets evolve as we use „selection“, „mutation“, „crossover“ and other methods. If we provide that starting „population“ become better and better on the end we will have optimal solution.

Using model based on input data related to GIS and algorithm for resolving p-median problem in combination and use that tool for optimization of Emergency medical service is common issue in literature. But, approach in problem definition and interpretation of results differ from case to case. Extensive studies dedicated to optimization of Emergency service in many towns are available. Such examles are studies for town Funen in Denmark [28], town Hyderabad in India [29] or Austin in Texas [30]. Different parameters taken in account, different source and structure of data, different goals of whole process in combination with everyday advance of technology, make these subject to be very interesting for researchers.

IV. Experimental work

“Emergency Medical Service” in town Niš, Serbia, is public service organized to meet a needs not only to residents of downtown, but also to residents of 68 neighborhood settlments. On surface about 600 km^2 there are more than 300.000 habitants and to succeed their needs service use 24 specialized ambulance vehicles and dozen vehicles with specific and unique medical equipement. Fleet is relatively new, well equiped with up-to-date standard ambulance medical and other units. Among them are units for satelite tracking and navigation. On duty are always 4 vehicles with complete stuff and thay cover shift 8 hours long. Stuff in one vehicle is composed from driver, nurse and doctor. After taking over a vehicle for shift they are available to dispecher in call-center and he accept requests for services and make decission who and when will go to serve the request. If there is no requests for service, vehicles are in central garage near emergency medical site and stuff is in leasure room waiting for call. All other vehicles are also in central garage in status of periodical service for recharging batteries, fuel filling, cleaning and washing etc. In addition to 4 vehicles on duty, there are 5 vehicles ready to be used in some unpredictable extraordinary situation. Stuff for these extra vehicles will be provided on different and case sensitive ways.

Assumptions we made in this paper begin with claim that every ambulance vehicle don`t need to wait in central garage, but had to be parked on parking location arranged to minimize average time of response. Every vehicle get coordinates of his parking location and after service go directly to that location. In cenrtal guarage comes on the end of the shift. Space on city map is devided into 4 regions and one region is attached to one vehicle. Parking locations for waiting can be determined in two different ways. One way is modeling of complete process and parameters and after testing of model through application make a prediction for probably the best solution. Quality of solution depends of quality of developed model. Another way is to analize recent period of time and find parking locations as best solution for already done services. Assumption is that if proposed location represent optimal solution for recent 30 days, that location will be optimal solution for next day also. We find optimum solution for real, already done using of vehicle, not for expected hipotetic using of vehicle in future.

One consequence of this approach is fact that in our experimental work appeared one location with extraordinary high weight factor and there is no patient on it. We investigate and find that on that location operate famous fast-food restaurant. It is wise to predict that in future time that location will remain as favourite destination for all ambulance stuffs. It is acceptable to leave this location in distance matrix with native weight factor and make optimization with this location involved. This kind of optimization have empiric caracter and reflect real life situations. On the other side, modeling and predicitons of events based on used models has his own advantages but also suffer according to high risk, because it is possible not to include some imortant factor in model. In that case we have poor quality of calculated solution.

V. Implemented algorithm decription

For purpose of resolving given problem algorithm in three phases is created. In first phase from database connected to GIS, coordinates of points of interest are extracted. In second phase starting set of medians is calculated using adjusted Teitz and Bart procedure. In third phase starting set of medians is improved using heuristic metod implemented with recalculations through several iterations. Adopted model of resolving p-median problem is adjusted to estimated maximum number of observed locations (n). It is sure that in period of 30 days one ambulace vehicle can serve only couple of hundred calls, and complete Emergency medical center fleet about one or two thousand interventions. Number of vehicles od duty (p) is also restricted. If number of interventions come to order of one hunderd thousend and more, proposed model is no more suitable and it is recomanded to implement another type of algorithm, for example some metaheuristic or aproximative.

In first phase of algorithm there is procedure for sequential search of database in which all data about moveing of ambulances are stored. Criteria for querry is defined and in implemanted subroutines we get answer on questions like: “Is vehicle moveing, or not?”. Based on these answers and predefined constraints, points which were destinations of the routes are extracted. After a first phase of algorithm we get n-locations as destinations in observed period of time. Results are sequentialy searched one more time and weight factor is defined. Criteria to join two demand nodes in one with weight factor of 2 is distance between them smaller than 60 meters. All numbers are converted in integer data type.

Second phase of algorithm on the flowchart on Fig.3 is represented. Aim of this part of the programme is to provide starting set of solutions. During third phase of the programme this set of solutions will be further improved. Key part of code in flowchart on Fig. 3 is the procedure named "1-median" and we continuously repeat that procedure. Purpose of the 1-median procedure is to perform required calculations on already pepared matrix and determine "first" median. On the begining of flowchart on Fig.3 we construct the matrix of distancies. We start with the basic assumption that any one demand node in pool containing observed n-demand nodes is qualified to be median. In other words, any observed location can be part of the final solution.

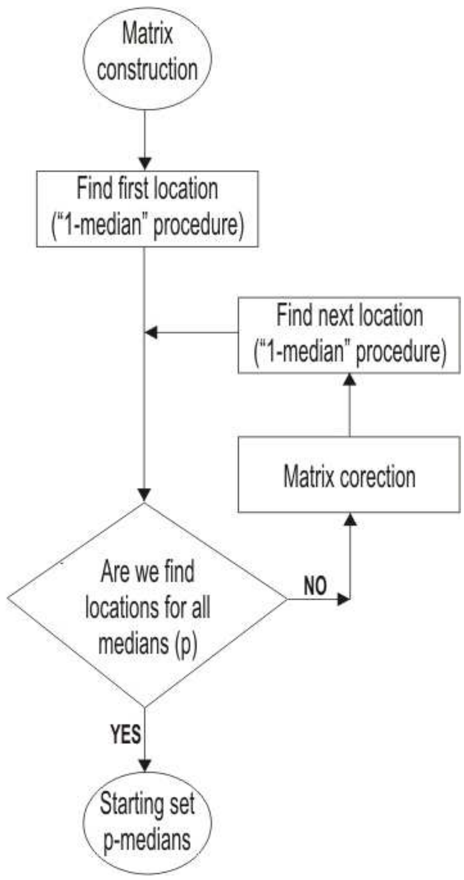

Fig. 3

We construct *n*x*n* matrix of distancies and introduce a weight factor. Matrix is consisted of *i*-rows and *j*-columns. Element d_{ij} represent distance from the demand node *i* to the parking location *j* multiplied by ω_1 - weight factor of node *j*. On this way formed matrix we apply 1-median procedure. In fact it is way to find the most suitable parking location if we have only one vehicle. For example we will take the third column of matrix. Third column is array of distancies to all other n - demand nodes if we start from the parking place located in node number “3”. Sum of all elements in our matrix placed in third column is total distance we had to travel if we want to serve all demand nodes and each time we start from the location in node number “3”. This sum is measure of the quality of location with number “3” and we use this sum to compare location to the other locations. We will name this sum as a “cost” of the location number “3”. In our case we made calculations using distancies, but the procedure is same in case we use time or fuel consumption needed to come in node *j* if you start from the node *i*. That is why name “cost” represent suitable term. We calculate sum for every column of the matrix and get cost related to each of n-nodes. Now we compare costs of the locations and find minimum. In hipotetic case that sum of all elements in column number “3” is accidentally minimum in comparision to other column sums, we have case, if we have just one vehicle, that vehicle should be parked on location defined with coordinates of the node “3”. It is the first median of our matrix. In the flowchart on Fig. 3 it is represented with first appearance of 1-median procedure. Of course, cost of the node number “3” will not remain unchanged through future calculations, because it will be influenced by introducing of the other medians. If we have no more vehicles, the second phase of algorithm should be finished here. However, as we have more vehicles we continue with the procedure to find next parking location. The first step is to made matrix correction. In the distance matrix we fill third row with zeroes. On this way we harmonize matrix according to the fact that if we start from any location and destination is node “3”, we need to travell zero kilometars. It is because we already have vehicle in node number “3”. On adjusted matrix we aply the “1-median” procedure again to find second median. On the flowchart on Fig. 3 this is represented through the branch returning to begining. We calculate sums of the matrix columns and we are looking for the minimum. The second median and cost of second location is obtained. We need to notice that cost of the first location is changed because we made matrix correction and the third row of the matrix ischanged. In our example we have four vehicles, so loop on Fig.3 is performed three times. When total number of the parking locations is exhausted, second phase of our algorithm is finished. As the result we have starting set of medians delivered through fast and simple procedure. On the end of this phase we sum all costs of delivered locations and get total cost of the whole solution. It will be used in to the future for comparision to the future proposed solutions. It is obvious that starting set of the medians is far from optimal, and we need lot of work to improve it.

The Fig. 4 shows flowchart implemented ih the third and final phase of our algorithm. This phase is dedicated to the improvement of the starting set of solutions. Distinctive procedure in third phase is the procedure named “neighborhooding”. "Neighborhooding" is process in which all nodes are devided in subsets and division is performed through finding of the nearest median for every node. As we have four medians in our example, we will divide all nodes in four subsets grouped around nearest median. We need to rearrange n-p nodes in total. On the flowchart on Fig. 4 this step is represented as apperance of the procedure “neighborhooding” for the first time. In that stage four subsets are created. Next step is application of the 1-median procedure but this time on every subset separately. 1-median procedure now deliver "local" minimum. It is node which represent local "first" median for observed subset. If comparision result in the findings that local minimum has smaller cost than previously founded median we made “vertex susbstitution”. So in the starting set of solutions we introduce local minimum instead of the starting median. Local minimum is now part of the final solution. We perform “neighborhooding” again and that is represented as the looping branch on the flowchart on Fig. 4. Loop is repeated until it is achieved that local minimum is equal or greater in comparision to corresponding median in the final set of solutions. We calculate cost of the whole solution and check if new iteration have lower cost as previous. Loop is repeated, and further “vertex susbstitution” and “neighborhooding” are performed. We continue until there is no more yield in cost of the solution set. As looping is ended we came to the optimal solution.

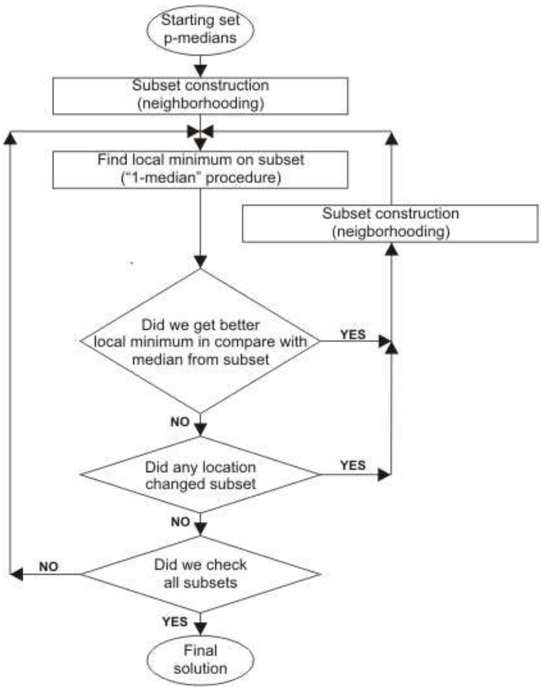

Fig. 4

Goal is acheived and we have the final set of locations for the parking places. Optimal rellocation of the vehicles is now possible. In same time we get subsets of the demand nodes and we have also resolved a problem of allocation. According to spatial position of the demand nodes in p-subsets we can split whole observed region in p-subregions. When new demand appear and coordinates of call are showed, dispecher in calling center can allocate proper vehicle in easy and quick manner. Spatial location of the subregions can be represented by suitable color representation on the GIS. On that way we contribute to better decision making proces.

On Fig. 5 we can see history and reailzed routes of one vehicle for period of 30 days as part of input data of our algorithm. On Fig. 6 we can see location of medians as final solution for parking places in our examle for 4 vehicles

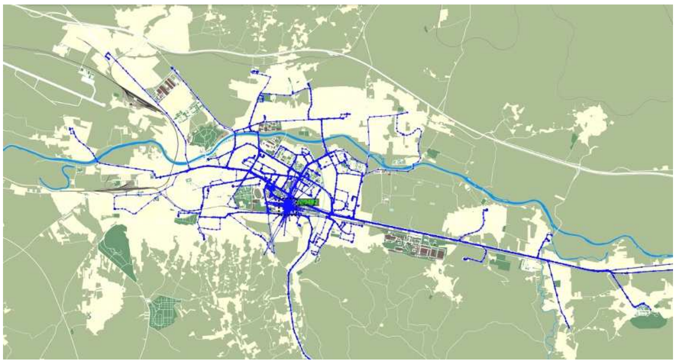

Fig. 5

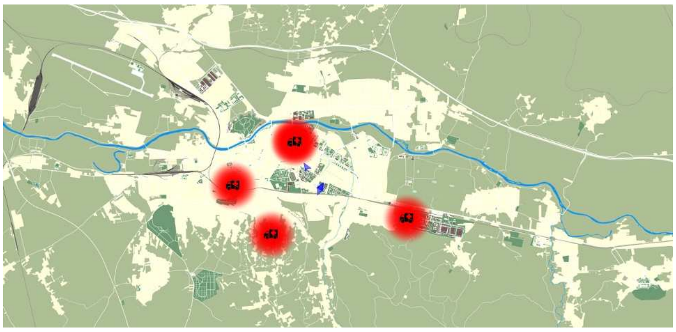

Fig. 6

VI. Conclusion

We justified claim that data collected using GIS open extraordinary posibilities for optimisation, can improve level of quality service and give convinient tool for better planing and saveings in everyday life. It is case in all fields of public servises and transportation of people and goods.

Practical results show that for resolving p-median problem in networks with about thousend nodes and dozen medians suitable tool represent modified Teitz and Bart method. Through process of verification and practical testing of results it is showed that for large amount average time to reach demand point was less if vehicle remain parked on proposed parking place to wait for call. Our solution is proposed to become a standard routine in Emergency Medical Service in Niš.

Further improvements should go in direction to incorporate an extra set of data in our algoritm related to GIS. It is set of data necessery for calculations in route planing and navigating vehicle to destination.

References:

[1] S. L. Hakimi. Optimum locations of switching centers and the absolute centers and mediansof a graph. Operations Research, 12(3):450–459, 1964.

[2] S. L. Hakimi. Optimum distribution of switching centers in a communication network andsome related graph theoretic problems. Operations Research, 13(3):462–475, 1965.

[3] C. ReVelle and R. Swain. Central facilities location. Geographical Analysis, 2:30–42, 1970.

[4] Kuehn and M. J. Hamburger. A heuristic program for locating warehouses. Management cience, 9(4):643–666, 1963.

[5] F. E. Maranzana. On the location of supply points to minimize transport costs. Operations esearch Quarterly, 15(3):261–270, 1964.

[6] M. B. Teitz and P. Bart. Heuristic methods for estimating the generalized vertex median of aweighted graph. Operations Research, 16(5):955–961, 1968.

[7] P. Hansen and N. Mladenovi´c. Variable neighborhood search for the p-median. Location Science, 5(4):207–226, 1997.

[8] F. Garc´ıa-L´opez, B. Meli´an-Batista, J. A. Moreno-P´erez, and J. M. Moreno-Vega. The parallel variable neighborhood search for the p-median problem. Journal of Heuristics, 8(3):375–388, 2002.

[9] T. G. Crainic, M. Gendreau, P. Hansen, and N. Mladenovi´c. Cooperative parallel variable neighborhood search for the p-median. Journal of Heuristics, 10(3):293–314, 2004.

[10] M. G. C. Resende and R. F. Werneck. A hybrid heuristic for the p-median problem. Journal of Heuristics, 10(1):59–88, 2004.

[11] K. E. Rosing and C. S. ReVelle. Heuristic concentration: Two stage solution construction. European Journal of Operational Research, 97(1):75–86, 1997.

[12] E. Rolland, D. A. Schilling, and J. R. Current. An efficient tabu search procedure for the p-median problem. European Journal of Operational Research, 96(2):329–342, 1996.

[13] O. Alp, E. Erkut, and Z. Drezner. An efficient genetic algorithm for the p-median problem. Annals of Operations Research, 122:21–42, 2003.

[14] L. A. N. Lorena and J. C. Furtado. Constructive genetic algorithm for clustering problems Evolutionary Computation, 9(3):309–328, 2001.

[15] M. Hosage and M. F. Goodchild. Discrete space location-allocation solutions from genetic algorithms. Annals of Operations Research, 6:35–46, 1986.

[16] S. Salhi. Defining tabu list size and aspiration criterion within tabu search methods. Computers and Operations Research, 29(1):67–86, 2002.

[17] G. Righini. A double annealing algorithm for discrete location/allocation problems. European Journal of Operational Research, 86(3):452–468, 1995.

[18] F. Chiyoshi and R. D. Galv˜ao. A statistical analysis of simulated annealing applied to the p-median problem. Annals of Operations Research, 96:61–74, 2000.

[19] G. Cornu´ejols, M. L. Fisher, and G. L. Nemhauser. Location of bank accounts to optimize float: an analytical study of exact and approximate algorithms. Management Science, 23:789–810, 1977.

[20] M. Hribar and M. S. Daskin. A dynamic programming heuristic for the p-median problem. European Journal of Operational Research, 101(3):499–508, 1997.

[21] M. B. Khumawala. An efficient branch-and-bound algorithm for the warehouse location problem. Management Science, 18(12):718–731, 1972.

[22] S. C. Narula, U. I. Ogbu, and H. M. Samuelsson. An algorithm for the p-median problem. Operations Research, 16(5):955–961, 1968.

[23] E. L. F. Senne and L. A. N. Lorena. Lagrangean/surrogate heuristics for p-median problems. In M. Laguna and J. Gonzalez-Velarde, editors, Computing Tools for Modeling, Optimization and Simulation: Interfaces in Computer Science and Operations Research, pages 115–130. Kluwer Academic Publishers, 2000.

[24] P. J. Densham and G. Rushton. Designing and implementing strategies for solving large location-allocation problems with heuristic methods. Technical Report 91-10, National Center for Geographic Information and Analysis, Buffalo, NY, 1991.

[25] R. A. Whitaker. A fast algorithm for the greedy interchange of large-scale clustering andmedian location problems. INFOR, 21(2):95–108, 1983.

[26] M. G. C. Resende and R. F. Werneck. On the implementation of a swap-based local search procedure for the p-median problem. In R. Ladner, editor, ALENEX ’03: Proceedings of the Fifth Workshop on Algorithm Engineering and Experiments, pages 119–127, Philadelphia, 2003. SIAM.

[27] T. H. Schietzelt and P. J. Densham. Spatial modelling, Chapter 17 Location-allocation in GIS, Cadcorp Ltd., Stevenage, United Kingdom, May 2001

[28] K. Lindeskov. Ambulance Allocation Using GIS. Master Thesis. Technical University of Denmark. August 2002

[29] Pasha. Ambulance management system using GIS. Master Thesis in Geoinformatics. Linkoping University. Sweden. May 2006

[30] N. M. Ruslim and N. A. Ghani. An Application of the p-Median Problem with Uncertainty in Demand in Emergency Medical Services. University Sains Malaysia, Penang. June 2006