ELECTROMECHANICAL ANALOGY AND GENERALIZED FUNCTION OF FRACTIONAL ORDER ENERGY DISSIPATION IN SPHERICAL NET DISCRETE CONTINUUM MODEL OF MOUSE ZONA PALUCIDA

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Abstract: During the process of maturation and fertilization Zona pelucida (ZP) passes through different oscillatory states. The aim of this work is to create an electrical analog model of fractional order spherical net model of mouse ZP (mZP) using electromechanical analogy. For that purpose the generalized Rayleigh function of fractional order energy dissipation and the extended system of Lagrange's equations of the second kind are considered. Several theorems of eigen main chains and energy changes are formulated.

Key words: zona pellucida, energy dissipation, fractional calculus, electro-mechanical analogy

1. Introduction

The analogy between mechanical and electrical systems is well known [1-3]. Electro-mechanical analogy can be used for parameterizations of shell-like deformations inside biological membranes [4]. Using discrete fractional order spherical net model of mouse Zona pelucida (mZP) it is possible to analyze energy of mZP and its dissipation before and after fertilization [5]. Adopting the methods presented in [1,3,5] we create an electro-mechanical analogy between one segment of mechanical fractional order spherical net model of mZP and its electrical counterpart (Fig 1). The dissipation of energy both in mechanical and electrical system is obtained. Dissipation of energy data is useful in developmental biology for bio-electro-mechanical explanation of the fertilization process. The practical value of this analogy is that electrical parameters can be measured and potentially applied in the diagnostics of reproductive pathology.

2. Electromechanical analogy and energy dissipation

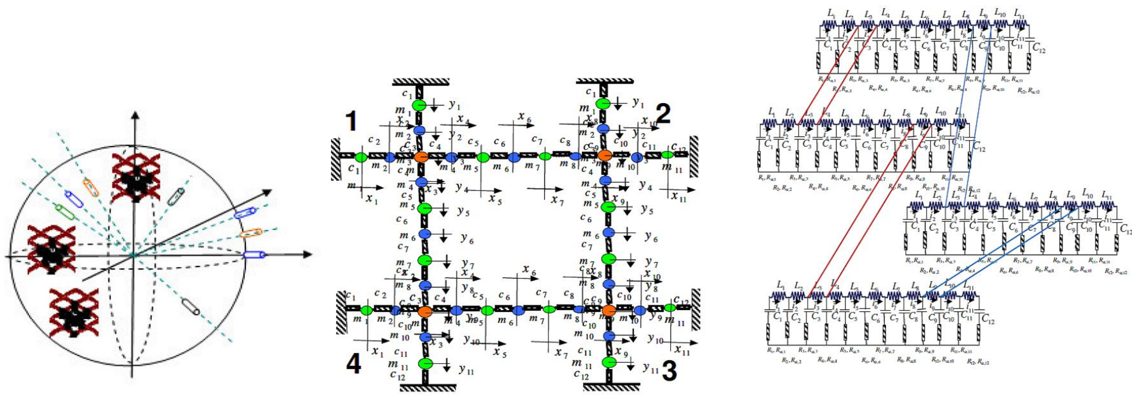
Using the generalized Rayleigh function of fractional order energy dissipation of a fractional order element, we create a generalized Rayleigh function of fractional order energy dissipation $\Phi_{chain,\alpha\neq 0}$, $0<\alpha\leq 1$, for one mechanical chain from representative part of the mZP net in the following form:

$$\Phi_{chain,0<\alpha<1}=\sum_{k=0}^{k=12}\frac{1}{2}c_{0<\alpha<1(k,k+1)}\left\langle \mathrm{D}_t^{\alpha}\left[x_{k+1}(t)-x_k(t)\right]\right\rangle^2,\ x_0(t)=0 \text{ and } x_{12}(t)=0,\ \alpha\neq 0,\ k=0,1,2,3,...,11,12. \quad (1)$$

Fractional order dissipation of electrical energy is analogous to fractional order dissipation of mechanical energy and is expressed by the corresponding generalized function of energy dissipation $\Phi_{chain,0<\alpha<1}$, $0<\alpha<1$. For one electrical chain the electrical analogue of the mZP net has the following form:

$$\Phi_{chain,0<\alpha<1} = \sum_{k=0}^{k=12} \frac{1}{2} R_{0<\alpha<1(k,k+1)} \left\langle \mathsf{D}_t^{\alpha}\left[q_{k+1}(t) - q_k(t)\right]\right\rangle^2 ,\ q_0(t)=0 \text{ and } q_{12}(t)=0,\ \alpha \neq 0,\ k=0,1,...12 .(2)$$

where $\mathsf{D}_t^{\alpha}[x(t)] = \frac{d^{\alpha}x(t)}{dt^{\alpha}} = x^{(\alpha)}(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{x(\tau)}{(t-\tau)^{\alpha}}d\tau$ is fractional order operator of the α^{th} derivative with respect to time t, $q_k(t)$ denote the electric charge analogous to coordinate displacement $x_k(t)$, $R_{0<\alpha<1(k,k+1)}$ is the coefficient of fractional order dissipation in electrical chain analogous to coefficient $c_{0<\alpha<1(k,k+1)}$ of fractional order damping forces.



a* b* c*

Fig.1 a**.** Part of the ZP spherical net model as a part of the sphere (oocite). Orange (ZP1), blue (ZP2) and green (ZP3) represent ZP glycoproteins. Net is identical in circular and meridian directions. An axis shows the direction of movements of ZP proteins. Each ZP protein is connected to the sphere. b. Segment of sphere surface net model of mZP: ZP glycoproteins are interconnected by standard light visco-elastic elements of fractional order (SLFOE): *m* –mass of the ZP glycoprotein, *c* – rigidity, *x* -displacements of ZP glycoproteins interconnected by SLFOE. in circular direction, *y* - displacements of mass particles interconnected of SLFOE in meridian direction. c. Electro-mechnical analogy between segment of oscillatory spherical net model of mZP as a mechanical oscillatory system with fractional order properties and a system of four electrical circuits with resistors with fractional order properties. *L*- self – inductance coils correspond to material particles, *C*- capacitator corresponds to spring in mechanical system, *R* – resistor in electrical circuits corresponds to dissipative element in mechanical system.

3. Conclusions

The total mechanical/electrical energy of a main mechanical/electrical chain decreases with time and is equal to the power of the fractional order force (electrical voltage) work during chain's fractional order oscillations. This electrical model can serve as theoretical background for studying the transition of electrical charge/voltage/current in ZP during process of maturation and after fertilization using electro–nano devices.

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