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ONE CLASS OF DESIGN AND THEIR APPLICATION IN THE EXPERIMENT ORGANIZATION

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ABSTRACT

The condition of the existance of one combinatorial configuration called design (block-schema) is in the base of determining the possibility of some experiment organization in many science discipline agriculture, biology,... and also in mining. With the design $B(v,r_1,\ldots r_v,b,k_1,\ldots,k_b,\lambda_{12},\ldots,\lambda_{v-1,v})$ as one combinatorial configuration can be presented organization of one experiment in them participate finite number v elements of some basic set, which should organise in the b design from defined number k_b elements from this base set but so that every of this elements are exactly in r_v designs and every pair of this different elements are in $\lambda_{v-1,v}$ designs.

In this paper is considered one "narrower" class of designs balanced incomplete block designs (BIBD) so called symmetrical BIBD which are important not only for the existance examining also for the construction of designs like second group of problems in the work with the designs. In the paper are given and one example of the application symmeetrical BIBD in mining experiment organization.

INTRODUCTION

Definition 1.1 Suppose that M is finite or infinite set. Every set of subsets, which consists elements of set M, is called configuration over the set M, it is marked with J and it is represented in form $=\{S_1,S_2,\ldots,S_m\}$(where every subset S_1 contains random number of elements). In the study of configurations three questions are essential: presentation, existence and construction.

Configuration can be graphically represented. We assign points of plane to the elements of the set $M = \{x_1, x_2,\ldots, x_n\}$ and every point, which belongs to set S_1, must be circled with a curve line. The configuration is given with the graph where the points of plane (like elements of set M) are connected with appropriate circle in planes that represent subsets S_i. Because of elaborated matrix-calculation the configuration is represented by so-called incident matrix. Suppose that $J=\{S_1,S_2,\ldots,S_m\}$ is configuration over set $M=\{x_1, x_2,\ldots, x_n\}$.For element x_j, $j=1,2,\ldots,n$, we say that it is in incidence with subset S_i $i=1,2,\ldots,m$, if $x_j \in S_i$.

Rectangular (0,1)-matrix $A=\{a_{ij}\}$, in form [m x n], whose elements are defined with

$$a_{ij} = \begin{cases} 1 & \text{if } x_j \in S_i \\ 0 & \text{if } x_j \notin S_i \end{cases} \qquad \forall i = 1,2,\ldots,m \wedge \forall j = 1,2,\ldots,n$$

is called incident matrix of the given configuration J over set M.

Because the configuration over set is given by subsets, because elements in those subsets are not organized and because schedule of this subsets in the configuration is not important we can say that different notices of elements in basic set, and also subsets in configuration, are possible. It leads us to the conclusion that one configuration can be corresponded by more incident matrix. The question is: in which form we should give

the configuration? The answer is very clear. We should change the configuration so that we get trivial form of incident matrix.

Definition 1.2 By design we allude any configuration $B=\{B_1,B_2,\ldots,B_b\}$ over finite set $V=\{a_1,a_2,\ldots,a_v\}$, where b i v are natural numbers. Design can be defined like definite pair (V,B) where $V=\{a_1,a_2,\ldots,a_v\}$, finite set of elements, and $B=\{B_1,B_2,\ldots,B_b\}$ set of subsets of different elements from V, or we can say set of blocks (we mean that $B_i \neq B_j$ for $i \neq j$).

Let us have design (V,B). For element a_j, $a_j \in V$ $j=1,2,\ldots,v$ we can say that it is incident to block B_i, $B_i \in B$ $i=1,2,\ldots,b$ if $a_j \in B_i$. With k_j, $j=1,2,\ldots,b$ we will mark total number of elements a_i, $a_i \in V$ $i=1,2,\ldots,v$ which are incident to block B_{-j}.

Total number of blocks B_j, $j=1,2,\ldots,b$, incident to element a_i, $i=1,2,\ldots,v$, we will mark with r_i. With $\lambda_{i\,t}$ we will mark the total number of elements of the set $\{B_j | a_i, a_t \in B_j\}$ for every $i=1,2,\ldots,v$ and $t=1,2,\ldots,v$, $i \neq j$. Because of undefinity the elements of the blocks we can say that $\lambda_{i\,t}=\lambda_{ti}$, so it is valid only to observe cases $i< t$. The numbers v, b, r_i, kj, $\lambda_{i\,t}$, are called argument of given design.

The use of this kind of configuration in solving the combinatory problems is very complicated. That is the way in which we will observe only balanced incompleted block designs marked with (v,r,b,k, λ) - configurations over finite set V, where the set V consists of v mutual different elements and configuration B is made of b blocks, every block is made of exactly k< v elements from V, and every element from V appears in exactly r<b blocks and every pair of different elements from V appears in exactly λ blocks.

It can be proved that if one (v,r,b,k, λ)-configuration over the finite set of elements exists, than two arguments are in the strict sdependence of permanent three, what will be explained in the following theorem which will be given without proof.

Theorem 1.1 If exists balanced design with arguments v,r,b,k,λ, over finite set of elements V i.e exists (v,r,b,k, λ) - configuration over finite set V, then the next equalities are correct

$$bk = vr \qquad (1.1)$$

and

$$r(k-1) = \lambda(v-1). \qquad (1.2)$$

Theorem 1.1 gives necessary but not enough conditions for existence of designs. Namely, if some of arguments v,r,b,k, λ satisfy relations from theorem 1.1, we are not sure that suitable configuration really exists: also, since the arguments are natural numbers, by giving some three, sometimes it is not possible to define permanent two only by using relations from theorem 1.1. The large number of testing in existence of designs has been performed, thanks to the evolution of the computers, depending on some arguments that could satisfy conditions from theorem 1.1 it is decided :

- for large values of argument v, balanced incompleted block design always exists and for small values it never does.
- for k = 3 and k = 4 theorem gives enough conditions for appropriate configuration, but not for k = 5.

Let us have some (v,r,b,k, λ)-configuration over finite set of elements V and we know its incident matrix $A=\{a_{ij}\}$, which is rectangular, in form [b x v].

Every hers column contains r units, and every hers rows contains k units. Scalar product of two mutual different vector-columns is equal to the number of appearance of the pair of different elements from V in configuration, namely equals λ. The scalar product of any vector-column with himself equals argument r, namely equals the number of appearance of any elements V in design. Those obvious attributes of incident matrix of (v,r,b,k, λ)-configuration, enable us to get necessary and enough conditions of its existents which are given with the next theorem:

Theorem 1.2 Rectangular (o,1) matrix $A=\{a_{ij}\}$, in form [b x v] is incident matrix of some (v,r,b,k, λ)-configuration over finite set of elements V. Then the next equalities are correct

(1.3) $$A^T A = (r - \lambda) I_v + \lambda J_v,$$

and

(1.4) $$A J_{v \times 1} = k J_{b \times 1}.$$

Vice versa is also correct.

Unfortunately, for determination of the arguments using the theorem 1.2, it is necessary to solve suitable matrix equality and that is not possible without corresponding mathematics method which is until now not developed.

Therefore we must take interest in other criterions, if they are not to complicated, for determination necessary and enough conditions of one design existance.

MAIN RESULTS

Definition 2.1 Let us have BIBD and next condition is valid

(2.1) $$b = v,$$

this BIBD is so called symmetrical balanced incomplete block design.

From (1.1) follow that is $k = r$, we can call this design (v,k,λ) – configuration, and from (1.2) is

(2.2) $$k(k-1) = \lambda(v-1).$$

Also, if rectangular (o,1) matrix $A=\{a_{ij}\}$, in form [b x v] is incident matrix of some (v,k, λ)-configuration over finite set of elements V, then the next equalities are correct

(2.3) $$A A^T = (k - \lambda) I_v + \lambda J_v,$$

and

(2.4) $$J_v A = k J_v.$$

We can prove that the necessary and enough conditions for existence of symmetrical designs are given with both relations (1.3) and (2.3) or (1.4) and (2.4) but for determination of the arguments using this relations, it is necessary to solve suitable matrix equality and that is not possible without corresponding mathematics method which is until now not developed. The next theorem gives necessary conditions for existance of symmetrical BIBDs :

Theorem 2.1 If exists one (v,k,λ) – configuration than

- if v is even number expression than k-λ is complete square some natural number;
- if v is odd number than so called Diofants equality have trivial solutions for x,y and z in the set of natural numbers:

(2.5) $$z^2 = (k - \lambda)x^2 + (-1)^{(v-1)/2} \lambda y^2.$$

Example 2.1 Let us examine if the experiment organization is possible in case of treatment a 15 new sorts of mines on 15 plants of grounds. Each plant of ground must be shared on 7 lots. The treatment must be organised so that each sort of mine be applied on 7 different plants of ground and each pair different sort of mine be applied three times in different plants of ground.

It is visible that solution of experiment possibility is in existance of balanced incomplete block-design BIBD with the next argument v=15,r=7,b=15,k=7, λ=3 i.e. {15,7,15,7,3}configuration over assembly V={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}:

B1={1,2,3,5,6,9,11}, B2={2,3,4,6,7,10,12}, B3={3,4,5,7,8,11,13},
B4={4,5,6,8,9,12,14}, B5={5,6,7,9,10,13}, B6={1,6,7,8,10,11,14},
B7={2,7,8,9,11,12,15},B8={1,3,8,9,10,12,13},B9={2,4,9,10,11,13,14},
B10={3,5,10,11,12,14,15},B11={1,4,6,11,12,13,15},B12={1,2,5,7,12,13,14},
B13={2,3,6,8,13,14,15},B14={1,3,4,7,9,14,15},B15={1,2,4,5,8,10,15}.

The solution of this configuration is possible using the following equalities :

from (2.1) b=v => 15 = 15 and

from (2.2) k(k-1)= λ(v-1) => 7*(7-1) = 3*14 => 42 = 42.

Necessary end enough conditions from (2.5) are also fulfilled. Diofants equation $z^2 = (k - \lambda)x^2 + (-1)^{(v-1)/2} \lambda y^2$ i.e. $z^2 = 4x^2 - 3y^2$ have a nontrivial solutions.

We can make construction one new design, v',r',b',k',λ' , using recursive method; Take one of B_i for example B_{15} ={1,2,4,5,8,10,15} and form a new Bi'

i =1,2,...,v-1=14 on the basis of equality $Bi' = B_i \cap B_v$ and in this way selected block

B_{15} take a place of basis set V and new disagn is formed as B={ B1', B2',..., B14'} with v'=k=7, r'=k-1=6, b'=v-1=14, k'= λ =3 and $\lambda = \lambda - 1 = 2$ i.e.

B1'={1,2,5}, B2'={2,4,10}, B3'={4,5,8},
B4'={4,5,8,}, B5'={5,10,15}, B6'={1,8,10},
B7'={2,8,15},B8'={1,8,10},B9'={2,4,10},
B10'={5,10,15}, B11'={1,4,15}, B12'={1,2,5},
B13'={2,8,15}, B14'={1,4,15}.

Because in the blocks of this design we have some equal and that B1'=B12',

B2'=B9',B3'=B4',B5'=B10',B6'=B8',B7'=B13' and B11'=B14' we have practically one {7,3,7,3,1} configuration.

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